SMI Based Beamforming Algorithms for TDMA Signals

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Abstract

This paper considers the problem of co-channel interference suppression in a TDMA communication system. To improve the reception of the signal of interest (SOI) and null interfering signals, the Sample Matrix Inversion (SMI) algorithm uses a known training sequence in each TDMA burst to tune the weights of an antenna array. However, when the interfering burst overlaps the SOI data but not the SOI training sequence, SMI performance is degraded. To overcome this problem, the Constant Modulus Algorithm (CMA), updated over the entire SOI burst, is used to enhance the SMI array weights. Simulations of a co-channel interference scenario demonstrate that SMI-CMA achieves higher output SINR than the SMI-Zero Forcing and SMI-Adjacent Burst algorithms for a large range of burst overlap cases.

1. Introduction

In a Time Division Multiple Access (TDMA) communication system, data is transmitted in bursts with a known training sequence occurring in each burst. This training sequence is designed to have good auto-correlation and cross-correlation properties for use in burst synchronization, burst identification, and equalizer training. However, in the presence of co-channel interference, a single-channel correlation of the received data with the training sequence will not yield a good correlation peak [4] [5] as shown in Figure 4.

An antenna array can be used to improve the reception of the signal of interest (SOI) and null the interfering bursts. An adaptive algorithm, known as the Sample Matrix Inversion (SMI) Method [1] locates and uses the block of array data containing the training sequence to tune the weights of the antenna array. A simplified block diagram is shown in Figure 1.

In situations where an interfering burst overlaps the SOI data but not the SOI training sequence, as shown in Figure 2, SMI performance is degraded. This occurs because SMI needs the temporal overlap of the SOI training sequence and the interference in order to direct an array null toward the interferer. Since SMI adaptation has no knowledge of Interfering Burst 2, the algorithm is unable to suppress the interference.

Figure 1. Simplified Beamformer Block Diagram

To overcome this problem, several methods to improve the SMI algorithm have been proposed. Leary and Gooch developed the SMI Zero Forcing (SMI-ZF) algorithm [4] which exploits the guard period on each side of the SOI burst, during which there is no SOI data transmission. The SMI-Adjacent Burst algorithm (SMI-Adj) [5] developed by Lindskog, uses the array data from the adjacent burst training sequence to improve the calculation of the beamformer weights. Figure 2 shows both the guard period, denoted as G, and the adjacent burst. Both algorithms rely on data outside of the SOI burst which contains the interfering burst signal. In this paper, the well-known Constant Modulus Algorithm [3][7], initialized with the SMI-ZF weights, is used to adapt the beamformer weights. The CMA adaptation is performed over the entire SOI burst and therefore is able to null interfering bursts that occur during the SOI data. Section 2 of this paper describes the adaptation algorithms. In Section 3, the performance of the pro-
posed algorithm is evaluated through simulations of a TDMA co-channel interference environment.

![Table showing Burst Overlap Diagram](image)

**Figure 2. Burst Overlap Diagram**

## 2. Adaptive Algorithms

For a narrowband $N$-element antenna array, the baseband received signal vector $\mathbf{x}(k)$ is given by

$$
\mathbf{x}(k) = \sum_{i=1}^{M} s_i(k) g_i(\theta) + \mathbf{n}(k)
$$

where $\mathbf{x}(k) = [x_1(k) \ x_2(k) \ldots x_N(k)]$ is a $1 \times N$ complex-valued vector and $k$ denotes discrete time. The co-channel transmitted signals are represented by $s_i(k)$, for $i = 1, \ldots, M$. The $1 \times N$ row vector, $g_i$, is the array response vector associated with the $i^{th}$ transmitted signal, which models the antenna array gain and phase across each of the elements. This is a function of angle-of-arrival, $\theta_i$ of the received signal. Noise is modeled by $\mathbf{n}(k) = [n_1(k) \ n_2(k) \ldots n_N(k)]^T$, a $1 \times N$ vector of complex white noise with variance $N_0$. The assumption is that each of the transmitted signals and noise sequences are mutually uncorrelated.

The sensor outputs are each multiplied by a complex weight $w_i(k)$ which may vary with time, and then summed to produce the output $y(k)$. The goal is to adjust the complex weights $w_i(k)$ to improve reception of the signal of interest (SOI). The array output is expressed as

$$
y(k) = \sum_{i=1}^{N} w_i(k) x_i(k) = \mathbf{x}(k) \mathbf{w}(k)
$$

where $\mathbf{w}(k)$ is the $N \times 1$ column vector of beamformer weights. The weight vector that minimizes the mean squared error $E[e^2(k)]$ is given by

$$
\mathbf{w}_{opt} = \mathbf{R}_{xx}^{-1} \mathbf{P}_{xd}
$$

where $\mathbf{R}_{xx} = E[\mathbf{x}(k)\mathbf{x}^H(k)]$ and $\mathbf{P}_{xd} = E[\mathbf{x}(k)\mathbf{d}(k)]$.

The Sample Matrix Inversion (SMI) [1] method is a technique used to approximate the solution to the MMSE problem. It assumes that there is a known training sequence $d(k)$ which occurs in the SOI data, that is, $s_j(k) = d(k)$ for some $j, k$.

First, $K$ samples of the signal vector $\mathbf{x}(k)$ are collected in a $K \times N$ matrix

$$
X_K(k) = \begin{bmatrix}
x_1(k) & \ldots & x_N(k) \\
x_1(k+1) & \ldots & x_N(k+1) \\
\vdots & \ddots & \vdots \\
x_1(k+K-1) & \ldots & x_N(k+K-1)
\end{bmatrix}
$$

This sample is used to form an estimate of the $N \times N$ covariance matrix

$$
\hat{\mathbf{R}}(k) = X_K^H(k)X_K(k)
$$

and $N \times 1$ cross-covariance vector

$$
\hat{\mathbf{P}}(k) = X_K^H(k)d(k)
$$

where

$$
d(k) = [d(k) \ d(k+1) \ldots d(k+K-1)]^T
$$

is a $K \times 1$ column vector.

The approximation to the solution of MMSE problem is calculated as

$$
\hat{\mathbf{w}}(k) = \hat{\mathbf{R}}(k)^{-1}\hat{\mathbf{P}}(k)
$$

SMI adaptation results in poor interference cancellation performance. This is due to the inadequate estimate of $\mathbf{R}_{xx}$ and $\mathbf{P}_{xd}$ using a finite size $K$ block of array data. Also, in an environment where signals are not continuously trans-
mitted, the problem of partial burst overlap of interferers further degrades SMI performance. This paper discusses three approaches to improving SMI performance.

In the first approach, Lindskog’s method, the data from the adjacent frame training sequence period is used. The reason is that a co-channel interferer which intersects the SOI data, but not the SOI training sequence, will be present during the training sequence of an adjacent frame.

The training sequence of the current frame is used to perform a system identification of the SOI array response vector $\hat{a}$ and the noise-plus-interferer (residual) signal vector $r(k)$. The same procedure is used to estimate the residual of the adjacent frame $r_{adj}(k)$. An estimate of the covariance matrices associated with the three vectors is summed to produce

$$ \hat{R}_{adj} = \hat{R}_{\hat{a}\hat{a}} + \hat{R}_{\hat{a}r} + \hat{R}_{r_{adj}r_{adj}} $$  \hspace{1cm} (9)

which is then used in SMI (8) to obtain the beamformer weights. See [5] for details.

The second approach, SMI-ZF, exploits the guard period on each side of the SOI burst during which there is no data transmission. Any interferer which does not intersect the SOI training sequence, but does intersect the SOI data, will intersect one of the guard periods. Hence, the training sequence vector in equation (7) becomes

$$ d_{tf}(k) = [0, d(k), d(k+1), \ldots, d(k+K-1), 0]^{T} $$  \hspace{1cm} (10)

where $0$ is a $l \times 1$ vector of zeroes and $l$ is the length of the guard periods. The sample covariance matrix in equation (4) is extended to include the corresponding samples of input array data $x(k)$.

The algorithm proposed in this paper uses CMA adaptation to improve the beamformer weights. SMI-ZF adaptation is used to locate the burst training sequence and provide the initial weights. After the initial weights are computed, the reference signal used in the CMA algorithm is

$$ d_{cma}(k) = \frac{\gamma(k)}{\|\gamma(k)\|}. $$  \hspace{1cm} (11)

CMA adaptation is used over the entire SOI burst. Decision directed adaptation of the beamformer weights could also be used [2][8], but the advantage of CMA adaptation is its simplicity and robustness.

### 3. Algorithm Performance

The performance of the proposed algorithm has been evaluated through simulations of a TDMA co-channel interference environment. A uniform linear array of eight antenna elements, spaced one-half wavelength apart, provides the input signals to the beamformer. The physical radio channel is assumed to be unity.

The modulation used is $\pi/4$ DQPSK, but the algorithm is applicable to other modulation types such as QPSK, DQPSK, MSK and GMSK. Note that although $\pi/4$ DQPSK signals do not satisfy the constant envelope property, the CMA algorithm can be used.

#### 3.1 Simulation 1

The first simulation uses an equal power co-channel interference signal, 17.4 dB SNR, arriving at an angle of $30^\circ$ relative to the SOI. The performance of the three adaptive algorithms is compared as the location of the interference burst is varied with respect to the start of the SOI burst. The performance is measured by calculating SINR at the output of the beamformer. For the transmitted signal $s_i(k)$, the SINR associated with the beamformer weights $w$ is calculated as

$$ SINR_i = \frac{\|a_i w\|^2}{\sum_{j \neq i} \|a_j w\|^2 + N_0^2 \|w\|^2} $$  \hspace{1cm} (12)

since the signal power is factored in $a_i$.

The simulations were performed using $4/3$ samples per symbol, however, Figure 3 is plotted against $2$ samples per symbol. As shown in Figure 3, the SMI-ZF-CMA algorithm achieves significantly higher output SINR than both the SMI-ZF and SMI-Adj algorithm for a large range of burst overlap cases. This 12 dB SINR performance improvement occurs when the interferer does intersect the training sequence and also when it does not. However, when there is less than 14 samples of burst overlap, the SMI-Adj algorithm achieves SINR which is 4 dB higher than SMI-ZF-CMA. In this situation, CMA adaptation over the burst does not have enough interference data to form a good estimate of $R_{xx}$. Consequently, the interference which overlaps the last few samples of SOI data is not nulled.
SMI beamforming, all seven co-channel signals can be detected.

**Figure 3. Simulation 1: Performance vs. Burst Overlap (One Interferer)**

### 3.2 Simulation 2

The performance of the SMI-ZF and SMI-ZF-CMA beamforming algorithms are compared in a second simulation where seven co-channel signals are present, as summarized in Table 1. The per element SNR, angle of arrival, and the input SINR of each signal is provided. The bursts are spaced ten samples apart.

**TABLE 1. Simulation 2: Scenario**

<table>
<thead>
<tr>
<th>Signal</th>
<th>SNR (dB)</th>
<th>Angle (degrees)</th>
<th>Input SINR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.8</td>
<td>90.0</td>
<td>-13.7</td>
</tr>
<tr>
<td>2</td>
<td>14.7</td>
<td>68.0</td>
<td>-24.0</td>
</tr>
<tr>
<td>3</td>
<td>18.7</td>
<td>86.9</td>
<td>-20.0</td>
</tr>
<tr>
<td>4</td>
<td>10.1</td>
<td>141.0</td>
<td>-28.6</td>
</tr>
<tr>
<td>5</td>
<td>18.6</td>
<td>174.3</td>
<td>-20.0</td>
</tr>
<tr>
<td>6</td>
<td>38.4</td>
<td>93.2</td>
<td>11.4</td>
</tr>
<tr>
<td>7</td>
<td>10.6</td>
<td>26.9</td>
<td>-28.1</td>
</tr>
</tbody>
</table>

As shown in Figure 4(a), a single sensor correlation with the training sequence only yields a clear correlation peak for signal 6, which has 11.4 dB input SINR. The remaining six signals can not be detected using single channel correlation. However, Figure 4(b) shows that with

**Figure 4. Simulation 2: Correlations**

(a) without beamforming
(b) with beamforming

For this scenario, a one hundred trial Monte Carlo simulation was performed. The results are summarized in Table 2. For comparison, the maximum attainable SINR for each signal is provided. The CMA adaptation raises the SMI-ZF SINR by up to 11 dB for some signals. Even in the presence of interfering bursts, the CMA adaptation does not lose lock on the signal of interest. The limited amount of data in one burst limits the SINR performance, but for signals with a maximum attainable SINR of at least 13 dB, SMI-ZF-CMA produces an output signal with at least 11 dB SINR. This results in a maximum bit error rate of $6 \times 10^{-3}$ for DQPSK signals in the presence of additive white gaussian noise and interferers, using a five-tap matched filter demodulator [6].

**TABLE 2. Simulation 2: SINR (in dB)**

<table>
<thead>
<tr>
<th>Signal</th>
<th>Max</th>
<th>SMI-ZF (mean)</th>
<th>SMI-ZF-CMA (mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.5</td>
<td>2.0</td>
<td>6.7</td>
</tr>
<tr>
<td>2</td>
<td>21.4</td>
<td>5.3</td>
<td>16.8</td>
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<tr>
<td>3</td>
<td>7.4</td>
<td>3.7</td>
<td>5.5</td>
</tr>
<tr>
<td>4</td>
<td>17.0</td>
<td>12.8</td>
<td>14.6</td>
</tr>
<tr>
<td>5</td>
<td>21.7</td>
<td>7.6</td>
<td>17.0</td>
</tr>
<tr>
<td>6</td>
<td>28.4</td>
<td>16.6</td>
<td>18.8</td>
</tr>
<tr>
<td>7</td>
<td>13.5</td>
<td>10.1</td>
<td>11.7</td>
</tr>
</tbody>
</table>
4. Conclusions

It has been shown through simulations that CMA adaptation can significantly improve the performance of the SMI algorithm in a TMDA co-channel interference problem. In a two interferer scenario, SMI-ZF-CMA achieves over 12 dB SINR improvement over SMI-ZF and SMI-Adj over a large range of burst overlap cases. In a seven interferer scenario, CMA adaptation achieves strong nulling of the interferers while maintaining lock on the signal of interest. Further work includes improving SMI burst detection in the presence of moderate and severe carrier offsets.

5. References


6. Acknowledgments

The authors thank Brian Sublett for providing technical discussions and support for this work; Cliff Prettie, Norm Yuen, and Newton Oku for carefully reading drafts.